Exercises set 02 – II. STATISTICAL CONCEPTS FOR UNCERTAINTY ESTIMATION

Part I

Exercise 2.1

Two sets of a dimensionless quantity measurements are performed:

Series A:

10.1 ; 9.9 ; 10.0 ; 10.2 ; 9.8

Series B:

10.5 ; 9.5 ; 10.7 ; 9.3 ; 10.0

- Calculate the mean of each series.
- Calculate the corresponding standard deviations.
- Which series is more accurate?
- Give the physical interpretation.

Exercise 2.2

A frequency (in Hz) is measured 40 times:

99.8 ; 100.2 ; 100.1 ; 99.9 ; 100.3 ; 99.7 ; 100.0 ; 100.4 ;
99.8 ; 100.1 ; 100.2 ; 99.9 ; 100.0 ; 100.3 ; 99.6 ; 100.2 ;
99.7 ; 100.1 ; 100.0 ; 99.8 ; 100.4 ; 100.2 ; 99.9 ; 100.1 ;
99.8 ; 100.3 ; 100.0 ; 99.7 ; 100.2 ; 100.1 ; 99.9 ; 100.0 ;
100.3 ; 99.8 ; 100.1 ; 100.2 ; 99.7 ; 100.0 ; 100.4 ; 99.9

- Determine the minimum, the maximum, and the range of the measurement series.
- Determine the optimal number of classes (justify your answer).
- Construct the grouped data table.
- Plot the histogram of values.
- Plot the cumulative histogram.
- Does the distribution appear to be normal?

Exercise 2.3

In an electronics laboratory, the noise of a stabilized voltage generator is studied. Fluctuations around the nominal value are due to many small independent

perturbations (thermal noise, vibrations, internal instability, etc.).
It is assumed that the instantaneous voltage follows a normal distribution:

$$X \sim N(5.00\ \text{V}; 0.02\ \text{V})$$

- What is the probability that the voltage lies between 4.98 V and 5.02 V?
- What is the probability that it exceeds 5.05 V?
- Determine the interval centered at μ containing 95% of the values.
- Physically interpret a result equal to 5.06 V.

Exercise 2.4

A) A length L is measured using an instrument. One hundred measurements are performed and the following results are obtained:

Mean: $\bar{l} = 12.007$ mm
Experimental standard deviation: $s = 0.024$ mm

- What is the probability distribution from which the values are assumed to to be derived or to be taken from?
- What is the degree of freedom ν of the measurement?
- Give a confidence interval for the mean at 68.27% confidence level, in the form:

$$L = \bar{l} \pm z_{theoretical} \cdot s$$

Specify the value of $z_{theoretical}$.

B) Another operator measures the same length L. The following 10 values (in mm) are obtained:
12.01 ; 11.98 ; 12.05 ; 12.00 ; 11.99 ; 12.03 ; 12.02 ; 11.97 ; 12.03 ; 11.99

- Calculate the experimental mean.
- Calculate the experimental variance s^2.
- Calculate the experimental standard deviation s.
- What is the degree of freedom ν?
- In this case, which probability distribution is appropriate to estimate the mean for a given confidence level?
- Give a confidence interval at 68.27% in the form:

$$T = \bar{T} \pm t_{theoretical} \cdot s$$

Specify the probability distribution used and the value of $t_{theoretical}$.

Compare the results obtained in A and B.

Exercise 2.5

The period T of a pendulum (in s) is measured 10 times. The measured values are: 2.01 ; 1.98 ; 2.03 ; 2.00 ; 2.02 ; 1.99 ; 2.01 ; 2.04 ; 1.97 ; 2.00

- Calculate the mean value $\bar{T}$.
- Calculate the experimental standard deviation s.
- What is the degree of freedom ν?
- Give a 95.45% confidence interval in the form:

$$T = \bar{T} \pm t_{theoretical} \cdot s$$

Specify the probability distribution used and the value of $t_{theoretical}$.

- The reference value is 2.00 s. Is the experiment compatible?

Exercise 2.6

A resistance is measured 8 times:

$$\bar{R} = 120\ \Omega$$
$$s = 6\ \Omega$$

Measurements are assumed to follow a normal distribution.

- Why should Student's distribution be used?
- Determine the degrees of freedom.
- Calculate the 95% confidence interval.
- Compare with the case where σ is known and equal to 6 Ω.
- Explain why the Student interval is wider.

Exercise 2.7

A dimensionless quantity is measured experimentally from 7 independent experiments. The results are:

$$\bar{x} = 12.4$$
$$s = 1.8$$
$$n = 7$$

The reference value is 11.
Measurements are assumed to follow a normal distribution.

- Determine the degree of freedom.
- Why must Student's distribution be used?
- Calculate the 95% confidence interval.
- Is the experiment compatible with the reference value?
- Give a physical (not only statistical) interpretation.

Exercise 2.8

A temperature sensor provides measurements characterized by a standard deviation:

$$\sigma = 0.5°C$$

Thirty-six independent measurements are performed. The obtained values mean is:

$$\bar{\theta} = 25.4°C$$

Random errors are assumed to follow a normal distribution.

1. Determine the 95% confidence interval.
2. Give the result in the form:

$$\theta = \bar{\theta} \pm U(\theta)$$

3. Physically interpret this confidence interval.
4. What happens if $n = 100$?

Exercise 2.9

Two groups of operators measure the same dimensionless quantity. They obtain:
Group A:

$$\bar{x} = 100, \qquad s = 4, \qquad n = 25$$

Group B:

$$\bar{x} = 100, \qquad s = 4, \qquad n = 9$$

1. Calculate the 95% confidence intervals.
2. Compare their widths.
3. Which group is more accurate?
4. Why?

Exercise 2.10

A resistor is measured 10 times. The results are:

Mean = 100 Ω

Experimental standard deviation = 2 Ω

1. Which test should be used?
2. Determine ν.
3. Calculate the 95% confidence interval.
4. Why is the interval wider than in the normal case?